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Non-linear Kalman filters for tracking a magnetic dipole

Marius Birsan

Defence R&D Canada

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Abstract

The problem of tracking a vessel modelled at a distance by an equivalent magnetic dipole is investigated. Tracking a magnetic dipole from magnetic field measurements is a complex non-linear problem. The determination of target position, velocity and magnetic moment is formulated as an optimal stochastic estimation problem, which could be solved using the non-linear Kalman filtering methods. The estimation performance of the following non-linear filters is compared: the extended Kalman filter (EKF), two Kalman filters based on the Stirling's interpolation formula, and the unscented Kalman filter (UKF). To evaluate the algorithms, the theoretical Cramer-Rao lower bounds (CRLB) of estimation error are derived for this problem. Results obtained on simulated track show that the target position, velocity and moment can be accurately determined. It is shown that the unscented Kalman filter (UKF) is the best non-linear Kalman filter for this application.

Résumé

Nous avons étudié le problème de la poursuite, à distance, d'un navire modélisé par un dipôle magnétique équivalent. La poursuite d'un dipôle magnétique, à partir des mesures du champ magnétique, est un problème non linéaire complexe. La détermination de la position, de la vitesse et du moment magnétique de la cible est formulée comme un problème d'estimation optimale aléatoire, que l'on peut résoudre en recourrant à des filtres de Kalman non linéaires. Nous avons comparé le rendement, sur le plan de l'estimation, des filtres non linéaires suivants : le filtre de Kalman élargi, deux filtres de Kalman basés sur la formule d'interpolation de Stirling et l'estimateur de Julier et Uhlmann. Pour évaluer ces algorithmes, nous avons utilisé, pour ce problème, les bornes théoriques inférieures de Cramér-Rao pour l'incertitude de l'estimation. Les résultats calculés à partir d'une trajectoire simulée montrent que l'on peut déterminer précisément la position, la vitesse et le moment magnétique de la cible. Nous montrons que l'estimateur de Julier et Uhlmann est le filtre de Kalman non linéaire le plus approprié pour cette application.

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Executive summary

Introduction

A requirement exists to improve the Navy's capability to conduct anti-submarine warfare (ASW) and intelligence, surveillance and reconnaissance (ISR) operations in littoral waters. To address this problem, a technology demonstration project (TDP) was set up to develop a Rapid Deployable System (RDS). The RDS concept includes an array of acoustic and magnetic sensors, communication links using radio frequency buoys and underwater acoustic modems, the necessary hardware, and automated (in-node) signal processing of the acoustic and magnetic data. Once the data was collected and processed, the transmitted information would allow the detection, localization and classification of any vessel navigating in the area. The magnetic information was added to the acoustic information to decrease the probability of false alarm and to aid the classification of the ship. Therefore, one of the objectives of the automated signal-processing task in the RDS project is to develop, test and integrate algorithms and software that improve the detection, localization and classification of the target making use of the magnetic data. The work described in this report is part of the development of the RDS automated signal processing capability.

The non-acoustic methods, especially magnetic, for ship's detection, tracking and classification have assumed greater importance in underwater applications where the acoustic methods give poor results, like in shallow waters, surf zones, and coastal regions. A vessel containing ferromagnetic material induces locally a magnetic disturbance (anomaly) with respect to the Earth's magnetic field. The anomaly is known as the magnetic signature of the vessel. When the vessel moves its magnetic signature as measured by a fixed sensor varies with time. Of interest is the determination of the trajectory and magnetic parameters of the vessel from the magnetic signature measurements, and one approach to this problem is investigated in this report.

Work description and results

Previous experience has indicated that the Kalman filtering technique can estimate in real time the motion and magnetic parameters of a vessel modeled by an equivalent magnetic dipole. In Kalman filtering, the tracking problem is formulated in state-space form where the state variables are the position, velocity and magnetic moment of the target. The relationship between the magnetic measurements and the motion and magnetic parameters of the target is non-linear. Lately, several filtering algorithms became available for state estimation of non-linear systems. These filters are the extended Kalman filter (EKF), two Kalman filters based on the Stirling's interpolation formula, and the unscented Kalman filter (UKF). They use various approximations of non-linear equation and thus their performances in solving the tracking problem will be different. The goal of this report is to investigate the non-linear estimators for the magnetic dipole tracking application and compare their performances.

It can be shown that the linear Kalman filter offers an optimal solution, so that no algorithm can do better. All non-linear filters are sub-optimal algorithms being based on approximations of state distribution. From this point of view, it would be important to have a theoretical lower bound of the best achievable error performance for this problem. The Cramer-Rao lower bound (CRLB) used in this study is convenient for evaluation purposes

and can also be used to determine the performance level that can be reached for the estimation problem. The filters were exercised on a set of simulated data and the absolute RMS error was used as the performance evaluation criteria. For the four filters, the RMS errors were obtained from 50 Monte Carlo runs with different Gaussian noise added, and compared with the theoretical CRLB. It is shown that the best magnetic signal processor for this application is the unscented Kalman filter (UKF).

Significance and future work

The need for detection under realistic conditions has motivated a series of trials where magnetic signals from various ships were recorded. The application of the presented algorithms to the experimental data indicated the possibility to incorporate them into the Rapid Deployable System TD project.

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Sommaire

Introduction

La Marine doit améliorer sa capacité de guerre anti-sous-marine, ainsi que ses activités de renseignement, de surveillance et de reconnaissance dans les eaux côtières. Pour aborder cette difficulté, on a créé un projet de démonstration technologique visant l'élaboration d'un système à déploiement rapide (SDR). Dans sa conception, ce système comporte un réseau de capteurs acoustiques et magnétiques, des liaisons informatiques assurées par des bouées radio-émettrices et des modems acoustiques sous-marins, le matériel nécessaire et le traitement automatisé (par les nœuds) des données acoustiques et magnétiques. Une fois les données captées et traitées, les informations transmises permettraient de détecter, localiser et classer tout navire qui croise dans la région. Le recours aux données magnétiques et acoustiques diminue la probabilité de fausses alertes et permet de mieux distinguer les navires. Ainsi, l'un des objectifs de la tâche automatique de traitement des signaux du projet SDR est d'élaborer, d'éprouver et d'intégrer les algorithmes et les logiciels qui utiliseront les données magnétiques pour améliorer la détection, la localisation et le classement des cibles. Le travail décrit dans ce rapport est une contribution à la mise au point des capacités de traitement automatisé des signaux du projet SDR.

On utilise de plus en plus les méthodes sous-marines non acoustiques pour la détection, la poursuite et la classification des navires, en particulier les méthodes magnétiques, lorsque les méthodes acoustiques ne donnent pas de bons résultats, notamment dans les eaux maigres, la zone des brisants et la zone côtière. Un navire qui comporte des matériaux ferromagnétiques induit une perturbation locale (une anomalie) du champ magnétique terrestre. On appelle cette anomalie : signature magnétique du navire. Puisque la signature magnétique d'un navire en mouvement, mesurée par un capteur fixe varie dans le temps, il serait intéressant d'en déterminer la trajectoire et les paramètres magnétiques, à partir de ces mesures. Nous présentons ici une méthode pour atteindre cet objectif.

Description du travail et des résultats

Des expériences antérieures indiquent que grâce aux filtres de Kalman, on peut estimer, en temps réel, le mouvement et les paramètres magnétiques d'un vaisseau modélisé par un dipôle magnétique équivalent. Pour utiliser les filtres de Kalman, on formule le problème de poursuite dans un espace-état dont les variables sont la position, la vitesse et le moment magnétique de la cible. La relation entre les mesures magnétiques, le mouvement et les paramètres physiques de la cible est non linéaire. Récemment, quelques algorithmes de filtrage permettant d'estimer l'état de systèmes non linéaires sont devenus disponibles : le filtre de Kalman élargi, deux filtres de Kalman basés sur la formule d'interpolation de Stirling et l'estimateur de Julier et Uhlmann. Puisque ces algorithmes utilisent diverses approximations de l'équation non linéaire, leur solution du problème différera. Nous décrivons ici notre recherche sur les estimateurs non linéaires utilisés pour suivre un dipôle magnétique et nous comparons leur rendement.

On peut démontrer que le filtre de Kalman linéaire donne la solution optimale et, donc, que son rendement est supérieur à celui des autres algorithmes. Tous les filtres non linéaires sont des algorithmes sous-optimaux qui reposent sur des approximations de la distribution d'état. De ce point de vue, il serait important de connaître pour ce problème, la borne inférieure théorique du meilleur rendement du point de vue de l'incertitude. Pour nos évaluations, nous avons utilisé la borne inférieure de Cramér-Rao, car elle est d'un emploi facile et on peut aussi l'utiliser pour déterminer quel niveau de rendement peut être atteint pour le problème d'estimation. On a appliqué ces filtres à une série de données simulées, en utilisant l'incertitude quadratique absolue comme critère d'évaluation du rendement. Pour les quatre filtres, nous avons calculé l'incertitude quadratique à partir de 50 simulations Monte Carlo, avec différents bruits gaussiens, que nous avons comparée avec la borne inférieure théorique de Cramér-Rao. Nous montrons que l'estimateur de Julier et Uhlmann est le meilleur algorithme de traitement des signaux magnétiques pour cette application.

Importance des résultats et travaux à venir

La nécessité de détecter les navires dans des conditions réalistes a justifié une série d'essais au cours desquels on a enregistré les signaux magnétiques de différents navires. L'application des algorithmes présentés aux données expérimentales indique qu'on peut les inclure au projet de démonstration de la technologie.

Birsan, M. 2003. *Utilisation de filtres de Kalman non linéaires dans la poursuite des dipôles magnétiques*. RDDC Atlantique TM 2003-230. R & D pour la défense Canada – Atlantique.

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1. Introduction

Recent defence strategy requires the monitoring of the activity of surface and underwater vessels in a certain area. For this reason, a surveillance Rapid Deployable System (RDS) was conceived that includes an array of acoustic and magnetic sensors, communication links using radio frequency buoys and underwater acoustic modems, the necessary hardware, and automated (in-node) signal processing of the acoustic and magnetic data. The information provided by the magnetic signal can increase the detection-classification capability of the surveillance system and decrease the probability of false alarm. Thus, one of the objectives of the automated signal-processing task in the RDS project is to develop algorithms and software that allow target detection, localization and classification based on magnetic data, and to test their performances.

Because the areas cover the shallow waters and coastal regions where the acoustic propagation conditions are poor and noise levels are high, the new generation of surveillance devices, such as the RDS, considered a combination of acoustic and magnetic sensors. A vessel containing ferromagnetic material can be adequately modelled at a distance by an equivalent magnetic dipole moment. This magnetic target can be observed by means of sea-floor tri-axial magnetometers that measure the variation of the magnetic field components as a function of time as it passes. These variations are known as the magnetic signature of the vessel. Of interest is the inverse problem of the determination of the trajectory and magnetic parameters of the vessel from the magnetic signature. One approach to solve this problem makes use of the Kalman filtering technique.

Precise determination of ship motion parameters (position, velocity), and ship classification (based on its magnetic moment) are primary concerns in an automated surveillance system. For practical and economical reasons, it is impractical to measure each of these parameters separately using different types of sensors. More convenient is to record a set of measurements, magnetic in this case, and to estimate (the mean and variance of) non-measurable parameters on the basis of available information. Accordingly, the ship tracking and classification can be regarded as an optimal stochastic estimation problem where one seeks the probability distribution of the unknown parameters conditioned to the measured data.

This study is a continuation of the previous SACLANT report on magnetic dipole tracking using Kalman filters [1]. The problem is formulated in state-space form where the state variables are the position, velocity and magnetic moment of the target. The relationship between the magnetic measurements and the motion and magnetic parameters of the target is non-linear, and this aspect must be specifically addressed when using the Kalman filtering technique.

The minimum mean square error (MMSE) estimate of the state of the system may be computed using the Kalman filter. As originally introduced [2], the Kalman filter assumes a linear relationship for both system dynamics and observation equations, and a Gaussian distribution of the state vector. A Gaussian distribution can be represented by a mean and a covariance matrix. The Kalman filter propagates the first two moments of the distribution of the state vector recursively and has a distinctive predictor-corrector structure described in [1, 2]. Subsequent extensions to Kalman filtering allow for non-linear equations, but still preserving the Gaussian distribution assumption. The development of filters for non- Gaussian distributions is the subject of a future study.

By linearizing the non-linear function around the predicted state values, the extended Kalman filter (EKF) is proposed to solve the magnetic tracking problem in [1]. Because of its first-order approximation of Taylor series expansion, the EKF algorithm leads to poor representation of the non-linear functions and probability distributions of interest. As a result, this filter may introduce large estimation errors due to filter divergence. Other estimation techniques are available that are more sophisticated than the EKF, e.g. iteration, higher order filters, and statistical linearization. In general, these techniques improve estimation accuracy at the expense of a further complication in implementation and an increased computation time. For example, the iterated extended Kalman filter (IEKF) was used in [1] and proved more successful in estimating the states than the EKF. It makes use of the second-order derivatives to reduce the effect of the measurement function non-linearity, but because of its computational burden it is not used in this study. It was replaced by an equivalent filter.

Since the publication of the SACLANT report [1], other algorithms became available to state estimation of non-linear systems. The new mathematical solutions were developed to surpass the difficulties related to the EKF, which is only reliable for systems that are almost linear on the time scale of the update intervals. These new filters use polynomial approximations obtained with an interpolation formula and the unscented transformation for derivation of state estimators for non-linear systems. The estimators become more accurate than estimators based on Taylor approximations. Yet the implementation is significantly simpler as no explicit calculation of Jacobians are necessary.

It can be shown that the linear Kalman filter offers an optimal solution, so that no algorithm can do better in the Gaussian environment. All non-linear filters are sub-optimal algorithms being based on approximations of state distribution. From this point of view, it would be important to have a theoretical lower bound of the best achievable error performance for this problem. The Cramer-Rao lower bound used in this study is convenient for evaluation purposes and can also be used to determine the performance level that can be reached for the estimation problem. The goal of this report is to investigate the new set of estimators for the magnetic dipole tracking application and compare their performances.

2. Tracking and classification of a magnetic dipole

2.1 Dynamic state-space model

The tracking problem requires estimation of the state vector (target co-ordinates, velocity, magnetic moment) of a system that changes over time using a sequence of noisy measurements (observations) made on the system. Practical applications of interest are in the field of surveillance for defence. For the specific application regarding this study, the target dynamics is described by a linear equation while the system observation equation is highly non-linear.

We wish to estimate the states of a non-linear dynamic system of the form:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}\left(\mathbf{x}_k, \mathbf{u}_k, \mathbf{v}_k\right) \\ \mathbf{z}_k &= \mathbf{h}\left(\mathbf{x}_k, \mathbf{u}_k\right) + \mathbf{w}_k \end{aligned} \tag{1}$$

The first equation in (1) represents the process model, while the second one represents the measurement (observation) model. In (1), $\mathbf{x}_k$ is the n_x-dimensional state vector of the system at time step k, $\mathbf{u}_k$ is the input vector, $\mathbf{z}_k$ is the n_z-dimensional observation vector, and $\mathbf{v}_k$ and $\mathbf{w}_k$ are vectors representing the process and measurement noise, respectively. They have the dimensions n_v and n_w. It is assumed that the noise vectors are i.i.d. and independent of current and past states. They are zero-mean and have the covariance estimates:

$$\begin{aligned} E\left[\mathbf{v}_i \mathbf{v}_j^T\right] &= \delta_{ij}\mathbf{Q}(i) \\ E\left[\mathbf{w}_i \mathbf{w}_j^T\right] &= \delta_{ij}\mathbf{R}(i) \\ E\left[\mathbf{v}_i \mathbf{w}_j^T\right] &= 0 \end{aligned} \tag{2}$$

The notation $\hat{\mathbf{y}}_{i\,|\,j}$ represents the estimate of $\mathbf{y}_i$ using the observation information up to and including time j, $\mathbf{z}^j = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_j\}^T$. Here T stands for transpose. The covariance of this estimate is $\mathbf{P}_{i\,|\,j}$. First, the filter updates the state of the system given an estimate $\mathbf{x}^{\wedge}_{k|k}$. The problem is solved for the a priori state and covariance estimates conditional on the past measurements up to and including time k:

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k} &= E\left[\,\mathbf{f}\left[\mathbf{x}_k, \mathbf{u}_k, \mathbf{v}_k\right] \,|\, \mathbf{z}^k\right] \\ \mathbf{P}_{k+1|k} &= E\left[\left(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k}\right)\left(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k}\right)^T \,|\, \mathbf{z}^k\right] \end{aligned} \tag{3}$$

If $\mathbf{f}(\bullet)$ and $\mathbf{h}(\bullet)$ are linear functions, and if Gaussian distributions are assumed for $\mathbf{x}$, $\mathbf{v}$, and $\mathbf{w}$, the estimation of states is reduced to the well-known Kalman filter [2] where the estimate $\mathbf{x}^{\wedge}_{k+1|k+1}$ is given by updating the prediction with the current measurements:

$$\begin{aligned} \mathbf{K}_{k+1} &= \mathbf{P}^{XZ}_{k+1|k}\left[\mathbf{P}^{ZZ}_{k+1|k}\right]^{-1} \\ \hat{\mathbf{x}}_{k+1|k+1} &= \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_{k+1}\left[\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right] \end{aligned} \tag{4}$$

where:

$$\hat{\mathbf{z}}_{k+1|k} = E\left[\mathbf{z}_{k+1} \mid \mathbf{z}^k\right]$$
$$\mathbf{P}^{XZ}_{k+1|k} = E\left[\left(\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k}\right)\left(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right)^T \mid \mathbf{z}^k\right] \tag{5}$$
$$\mathbf{P}^{ZZ}_{k+1|k} = E\left[\left(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right)\left(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right)^T \mid \mathbf{z}^k\right]$$

The update of the covariance matrix is:

$$\mathbf{P}_{k|k} = E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right)^T \mid \mathbf{z}^k\right] = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}^{ZZ}_{k|k-1} \mathbf{K}^T_k \tag{6}$$

Because the above equations are function only on the predicted values of the first two moments of $\mathbf{x}_k$ and $\mathbf{z}_k$, the possibility of applying the Kalman filtering technique to non-linear systems consists in the ability to estimate these moments.

The problem of tracking a magnetic dipole does not satisfy the original Kalman filter requirements because the system observation is non-linear. In order to deal with non-linear systems, two categories of techniques have been developed and they are described in the literature [3, 4, 6]: parametric and non-parametric. The parametric techniques are based on improvements in linearizing the equations of the Kalman filter and make the subject of the present study. The non-parametric techniques are based on Monte Carlo simulations and will be presented in a future study.

2.2 Magnetic dipole dynamics and measurements

For the tracking and classification problem under investigation, the mathematical model used for the target is a moving magnetic dipole. The model appears to be adequate for practical purposes if the distance between the real target (vessel) and sensor is relatively large with respect to the overall dimension of the target. At short ranges, where more complex modelling is necessary to describe the spatial distribution of magnetic sources, this model is a simplification. The target is fully characterised by its motion parameters (position and velocity) and the value of the magnetic dipole moment. When all these parameters are known the target may be classified as a surface or submerged vessel having a certain mass, which can be appreciated after the amount of ferromagnetic material contained.

Ship motion is rather slow and could be described using a constant velocity model. This mathematical model is sufficient to provide satisfactory estimation accuracy under the condition that the target does not accelerate during the observation time. The constant velocity assumption is valid because the magnetic signal is caused by a short-range effect being observable for a short period of time when the target is close to the sensor. During this period it is less probable that the target undergoes abrupt manoeuvres. Also the magnetic mass of the target remains constant during the passage and can be estimated from the values of the equivalent magnetic dipole moment.

Let consider that the time increment between the data samples is Δt in seconds, $\mathbf{m}$ is the magnetic moment vector of the dipole expressed in 10^6 A•m^2, $\mathbf{V}$ is the velocity vector in m/sec, and $\mathbf{r}$ is the position vector from the observation point to the dipole centre in meters.

For a full characterisation of the target, the entire system at time step k can be represented by the state vector:

$$\mathbf{x}_k = \left(r_X \; r_Y \; r_Z \; V_X \; V_Y \; V_Z \; m_X \; m_Y \; m_Z\right)^T \tag{7}$$

The discrete equations of target motion are obtained using the piece-wise approximation:

$$\begin{aligned}
r_X(k) &= r_X(k-1) + \Delta t \, V_X \\
r_Y(k) &= r_Y(k-1) + \Delta t \, V_Y \\
r_Z(k) &= r_Z(k-1) + \Delta t \, V_Z \\
\\
V_X(k) &= V_X(k-1) \\
V_Y(k) &= V_Y(k-1) \\
V_Z(k) &= V_Z(k-1) \\
\\
m_X(k) &= m_X(k-1) \\
m_Y(k) &= m_Y(k-1) \\
m_Z(k) &= m_Z(k-1)
\end{aligned} \tag{8}$$

Tri-axial magnetic sensors produce the observation data. The magnetic flux density vector **B** at a given point due to a magnetic dipole is given by the formula:

$$\mathbf{B} = \frac{\mu}{4\pi}\left[3\langle \mathbf{r}, \mathbf{m}\rangle \mathbf{r} - |\mathbf{r}|^2 \mathbf{m}\right] / |\mathbf{r}|^5 \tag{9}$$

where μ is the permeability of the medium (= $4\pi \; 10^{-7}$), and $\langle \bullet, \bullet \rangle$ is the inner product operator. By introducing the time variable to account for the movement of the target, the three magnetic flux components measured at the observation point at time step k are:

$$\begin{aligned}
B_x(k) &= \frac{\mu}{4\pi}\left[3 r_x(k) < \mathbf{r}(k), \mathbf{m} > - |\mathbf{r}(k)|^2 m_x\right] / |\mathbf{r}(k)|^5 \\
B_y(k) &= \frac{\mu}{4\pi}\left[3 r_y(k) < \mathbf{r}(k), \mathbf{m} > - |\mathbf{r}(k)|^2 m_y\right] / |\mathbf{r}(k)|^5 \\
B_z(k) &= \frac{\mu}{4\pi}\left[3 r_z(k) < \mathbf{r}(k), \mathbf{m} > - |\mathbf{r}(k)|^2 m_z\right] / |\mathbf{r}(k)|^5
\end{aligned} \tag{10}$$

For one sensor, the measurement vector at time k has the form:

$$\mathbf{z}_k = \left(B_x \quad B_y \quad B_z\right)^T \tag{11}$$

These are the process and measurement equations used by the Kalman filtering technique for tracking and classification of a magnetic dipole. As one can see, the process function $\mathbf{f}(\bullet)$ in equation (1) is linear, and the measurement function $\mathbf{h}(\bullet)$ is highly non-linear.

2.3 Kalman filters for non-linear systems

The purpose of this study is to implement four Kalman filters developed for non-linear systems and evaluate their performances when applied to solve the magnetic dipole-tracking and classification problem. The filters determine in an approximate way the mean and covariance matrix of the probability of target state conditioned to the measured magnetic data. These filters are the EKF, two Kalman filters based on the Stirling's first and second-order approximations (ST1 and ST2), and the unscented Kalman filter (UKF). All of them are based on approximations and our goal is to assess the influence of a particular approximation on the tracking accuracy. A brief description of the filters is given below and more details can be found in literature [3, 4, 6].

The extended Kalman filter (EKF) is a MMSE estimator based on the first-order Taylor approximations of state transition and observation equations about the estimated state trajectory. The result is that the EKF calculates the first two moments accurately to the first-order with all higher order moments truncated. A difficulty related to this filter is the need to calculate the Jacobian matrix of the $\mathbf{f}(\bullet)$ and/or $\mathbf{h}(\bullet)$ functions, whichever is non-linear. For example:

$$\mathbf{h}(\mathbf{x}_k) \cong \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}) + \frac{\partial \mathbf{h}(\mathbf{x}_k)}{\partial \mathbf{x}_k}\Big|_{(\mathbf{x}_k = \hat{\mathbf{x}}_{k|k-1})} (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}) + \ldots \tag{12}$$

Application of the filter is limited to the cases where the required derivatives exist and can be calculated with a reasonable effort. The implementation of the EKF is described in [1, 6].

Different from the Taylor approximation, the Stirling's interpolation formula of the non-linear transformations needs no derivatives, but only function evaluations. The idea was to replace the derivatives in the Taylor approximation by central differences:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2u}; \quad f''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{u^2} \tag{13}$$

where u denotes a selected interval length, which has an optimum value of $\sqrt{3}$ for a Gaussian distribution [4]. These formulas accommodate easy implementation of two filters based on the first (ST1) and second-order (ST2) approximations, respectively. The implementation is less complicated than for filters based on Taylor approximations because the calculation of the Jacobians is avoided and the computation time is comparable or only slightly bigger. Because the ST1 filter is derived by employing the first-order approximation, it corresponds to the EKF except that the Jacobians are replaced by differences. A similar observation applies for the ST2 filter, which is the equivalent of the IEKF [1] being both constructed using the second-order approximation.

The unscented Kalman filter (UKF) is based on the unscented transformation [3], which represents a method to calculate the first two moments (mean and covariance) of a random variable that undergoes a non-linear transformation. These estimates of the mean and covariance are accurate to the second order of the Taylor series expansion for any non-linear function. Errors are introduced in the third and higher order moments, but are scaled by the choice of a scaling parameter. The complete UKF algorithm that updates the mean and covariance of the distribution of the state variables is given in Appendix.

2.4 Posterior Cramer-Rao bound

The posterior Cramer-Rao bound gives the inferior limit for one step ahead prediction, which is the estimation at time step k of $\mathbf{x}_{k+1}$ given the measurements $\mathbf{z}^k = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k\}^T$ up to and including time k. As mentioned, the optimal estimator for the non-linear Gaussian tracking problem cannot be built, and hence we resort to approximate filters described above. The theoretical lower bound plays an important role in algorithm evaluation and assessment of the level of approximation introduced in the filtering algorithm.

The application of the Cramer-Rao bound to a sequential problem would require forming the joint probability $p(\mathbf{x}_{k+1|k}, \mathbf{z}^k)$. In the following, the notations will be used for convenience:

$$\begin{aligned} &\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1}, \\ &\mathbf{R}_k = \mathbf{R}(k), \quad \mathbf{Q}_k = \mathbf{Q}(k) \end{aligned} \tag{14}$$

The correlation matrix of the estimation error is at any time instant, k, limited from below by a matrix:

$$E\left\{(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^T\right\} \geq \mathbf{J}_k^{-1} \tag{15}$$

where $\mathbf{J}_k^{-1}$ is the posterior Cramer-Rao lower bound (CRLB), which defines the best achievable performance of the filtering algorithm. The information matrix $\mathbf{J}_k$ is defined:

$$\mathbf{J}_k = E\left\{\left[\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k, \mathbf{z}^k)\right]\left[\nabla_{\mathbf{x}_k} \log p(\mathbf{x}_k, \mathbf{z}^k)\right]^T\right\} \tag{16}$$

where $\nabla_{\mathbf{x}}$ denotes the gradient operator with respect to vector $\mathbf{x}$. The results are presented here without proofs or derivations, and the details can be found in [5]. For the case of additive Gaussian noise in equation (1), the Cramer-Rao lower bound for the one step ahead prediction of the states obeys the following recursion rule:

$$\mathbf{J}_{k+1}^{-1} = \mathbf{F}_k\left[\mathbf{J}_k + \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k\right]^{-1}\mathbf{F}_k^T + \mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^T \tag{17}$$

where:

$$\begin{aligned} \mathbf{F}_k^T &= E\left\{\nabla_{\mathbf{x}_k} \mathbf{f}^T(\mathbf{x}_k, \mathbf{v}_k)\right\} \\ \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k &= E\left\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \log p(\mathbf{z}_k \mid \mathbf{x}_k)\right\} \\ \mathbf{G}_k^T &= E\left\{\nabla_{\mathbf{v}_k} \mathbf{f}^T(\mathbf{x}_k, \mathbf{v}_k)\right\} \\ \mathbf{Q}_k^{-1} &= E\left\{-\Delta_{\mathbf{v}_k}^{\mathbf{v}_k} \log p(\mathbf{v}_k)\right\} \end{aligned} \tag{18}$$

In the above formula, the operator $\Delta_{\mathbf{x}}^{\mathbf{x}} = \nabla_{\mathbf{x}}\nabla_{\mathbf{x}}^{T}$ is the Laplacian. One can see that $\mathbf{F}_k$ and $\mathbf{H}_k$ are the Jacobians of $\mathbf{f}(\bullet)$ and $\mathbf{h}(\bullet)$ evaluated at the true state. The recursion (17) is initiated by:

$$\mathbf{J}_0^{-1} = E\left\{-\Delta_{\mathbf{x}_0}^{\mathbf{x}_0} \log p(\mathbf{x}_0)\right\} \tag{19}$$

The Cramer-Rao lower bound of an unbiased estimate of component j=1,2,...,n_x of state vector $\mathbf{x}(k)$ at time instant k is then:

$$CRLB\{x_k^j\} = \mathbf{J}_k^{-1}[j,j] \tag{20}$$

There are different ways to use the CRLB for recursive estimation [5]. In this work, the bounds are used for evaluation of sub-optimal algorithms by means of Monte Carlo simulations. The performance of the four Kalman filters is compared on the basis of their root mean square (RMS) errors. This error depends on the region of the state-space in which the simulation is carried out and on the noise levels used. By calculating the lower bound against which to compare the results, a measure of the effectiveness of each algorithm is obtained for that specific simulation study. For this reason the simulated scenario must be representative for the problem.

2.5 Simulated experiments and results

One of the problems encountered in designing an experiment is the system observability. A system is observable when its state vector can be reconstructed from the measurements of its output. In the practical problem discussed here, it is important to know in what conditions the measurement of the magnetic flux density will reveal all the information about the system state vector described by (7). Because the magnetic flux density depends on both states **m** and **r**, it is not possible to reconstruct these (six) states from a single sensor observation, which produces at most 3 data points. If a single sensor is used for observation, one must assume that some of the states in vectors **m** and/or **r** are known.

Two examples of this kind (1-observer) are presented in the SACLANT report [1] where the proposed state vectors are $\mathbf{x} = (r_x\ r_y\ r_z\ V_x\ V_y)^T$ and $\mathbf{x} = (r_x\ r_y\ V_x\ V_y\ m_x\ m_y)^T$, respectively. As one can see, in both cases the tracking and classification problem is only partially solved. Based on these two examples, the conclusion of the report [1] was that the EKF, which is constructed on the basis of a first-order approximation, is not satisfactory and the algorithm diverges. Also it is shown that the IEKF, which improves the linearization of the measurement equation by using the second-order derivatives in the Taylor series expansion, can perform successfully. Even so, the state variable estimates are biased and lack precision due to a reasonably large variance. However, the results presented in [1] proved that a better approximation of the non-linear equation could lead to a better performance of the tracking algorithm and the present work follows this direction of improvement of the Kalman filter.

In this work, the non-linear Kalman filtering technique previously [1] intended for a single observer situation, is extended to a 2-observers situation. The advantage is that the complete problem can be solved in the new situation, i.e. one can determine unambiguously the horizontal position versus time, depth, speed, and the equivalent dipole moment of the

moving target. In this case, the state vector is given by equation (7). This is a more complicated problem than the previous (1-observer) one where simplifying assumptions were required. The performance of a 9-state filter, where 2 observers are required to preserve the system observability, was not evaluated in the SACLANT report [1].

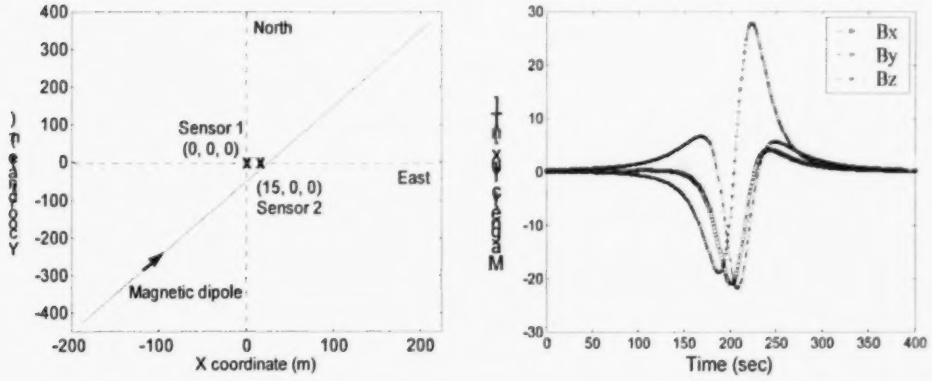

Figure 1. *Simulated track, sensor position, and magnetic field at sensor 1.*

Simulated observations are used in this study: a true trajectory is assumed for a magnetic dipole having a known moment, observations are computed using equation (10) and contaminated with Gaussian noise. The SNR is 10 dB for all simulations.

The scenario used to evaluate the filters is the same as the one used in the SACLANT report [1] and it is considered representative for the tracking problem. The target is represented by a magnetic dipole with a constant moment over time, $\mathbf{m} = (10^5 \;\; 10^5 \;\; 0)$ A•m^2. The target approaches the observers at a constant velocity, $\mathbf{V} = (1 \;\; 2 \;\; 0)$ m/sec, and at a distance from the bottom of 75m. **Table 1** gives points on the simulated track. The total travel distance of the target is 894.5m and the travel time is 400sec. Discrete observations (eq. 11) are taken at a sampling period of one second. Two tri-axial magnetometers (observers) are placed on the seafloor having the positions given by $\mathbf{s}_1 = (0 \; 0 \; 0)$ and $\mathbf{s}_2 = (15 \; 0 \; 0)$ meters, respectively. **Figure 1** shows the simulated track, the position of the sensors, and the observations at the first sensor, i.e. the magnetic flux density components in nano-Tesla [nT] versus time.

Table 1. *Points on the simulated track*

Trajectory points	Time (sec)	r_x (m)	r_y (m)
Start point	0.0	-187.5	-425.0
CPA (sensor 1)	207.5	20.0	-10.0
Stop point	400.0	212.5	375.0

The initial (t = 0 sec) noisy magnetic signal is very small, it could be at the environmental level, so that practically it is assumed that the time at which the signal occurs is unknown. From this point of view, the tracking problem can be considered a detection problem as well. Practically, the filters are started at t = 0 sec.

In applying the Kalman filter to the system, the initial conditions and the noise covariance matrixes need to be specified. The basic assumption is that the target can approach the sensors from any horizontal direction. Therefore, a reasonable initial estimate of the horizontal position is the null vector. For the vertical position, an initial estimate between zero and the approximate water depth can be given. Because we have no information about the magnitude of velocity and magnetic dipole moments, a good initial estimate of these vectors are merely the null vectors. For the 9-state, 2-observers filter exercise, the initial state estimate is:

$$\mathbf{x}_0 = \begin{pmatrix} 0 & 0 & r_Z & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The initial covariance matrix, $\mathbf{P}(0|0)$, gives a measure of belief in the initial state estimate. It is assumed that initially all the states are un-correlated, so that the matrix is diagonal. The initial estimates affect the transient performance of the algorithm and the choice of the appropriate values will prevent filter divergence. In this particular problem, small values for initial variances give large values of filter gain meaning that the initial observations are heavily weighted and the model is ignored. On the other hand, too large variance values make the filter gain extremely small and the algorithm diverges. These aspects are important because the initial covariance is unknown and need to be trimmed.

The measurement noise covariance matrix can be estimated directly from the actual data and, once calculated, it does not change during the Kalman filter run. The process noise covariance is zero for a deterministic process. However, it was practically proved to be a good idea to introduce random perturbations in the target position, velocity, and magnetic moment. These small perturbations account for the mis-modeling produced by linearization and prevent divergence, so that the process noise covariance may be regarded as a tuning parameter of the filter.

The comparison of the performance of the four algorithms is based on their Monte Carlo root-mean-square (RMS) error, which is commonly used in signal processing applications. The RMS error depends on the simulation conditions used in the Monte Carlo evaluation, such as the choice of noise levels and distributions. A measure of the relative effectiveness of each algorithm is obtained by comparing the results against the theoretical CRLB. The discrepancy of the absolute RMS error from this lower bound reveals the effects of the sub-optimal approximations introduced in the corresponding algorithm. The absolute RMS errors obtained after 50 Monte Carlo runs for the four filters are plotted versus the square root of theoretical CRLB in figure 2. Each Monte Carlo run has a different noise vector added to the observation vector.

It is a common practice in the tracking problems to start the filters at a point where a minimum error in the initial (horizontal) position estimates is achieved. This point would correspond to the CPA in the present case. One way to do this in practice is to wait until the magnitude of the magnetic flux density is at the maximum and then start the filters. Figure 3 presents the absolute RMS errors for this situation calculated from 50 Monte Carlo runs also. The Z-component of the magnetic moment is unchanged (zero) and is not shown. There is not a significant improvement in estimations because the rest of the state variables have the same initial estimation errors as in the previous case.

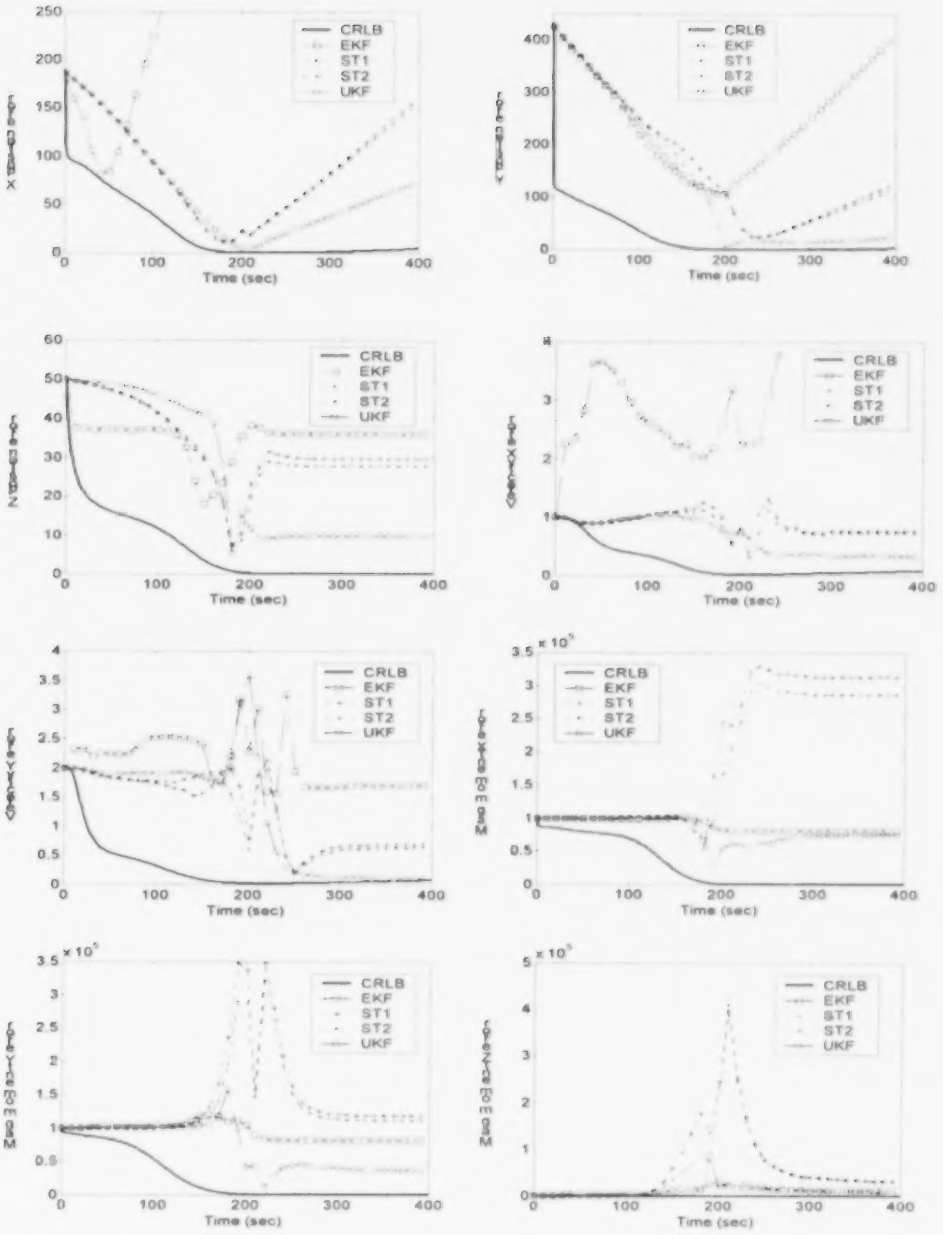

Figure 2. *Monte Carlo RMS errors and square-root of CRLB*

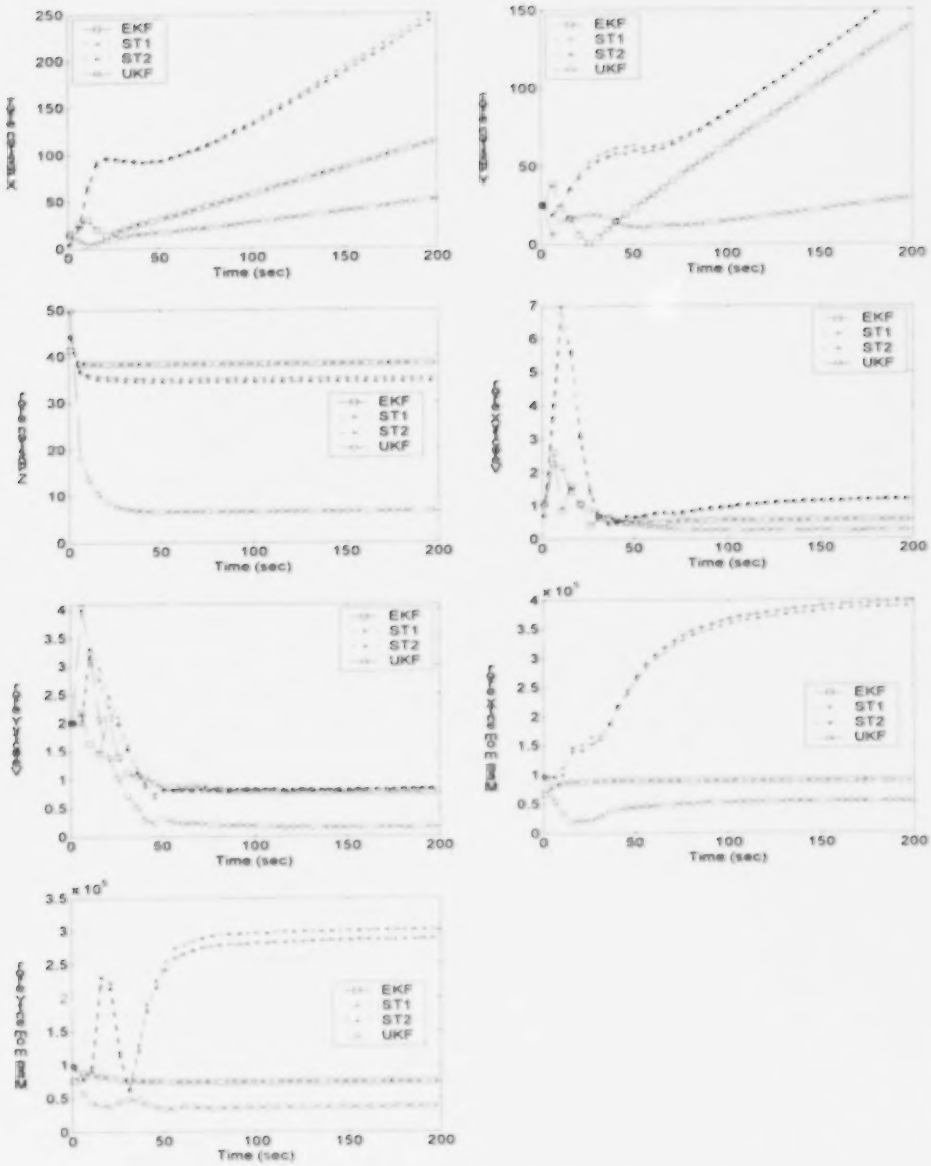

Figure 3. *Monte Carlo RMS errors for filters started at CPA*

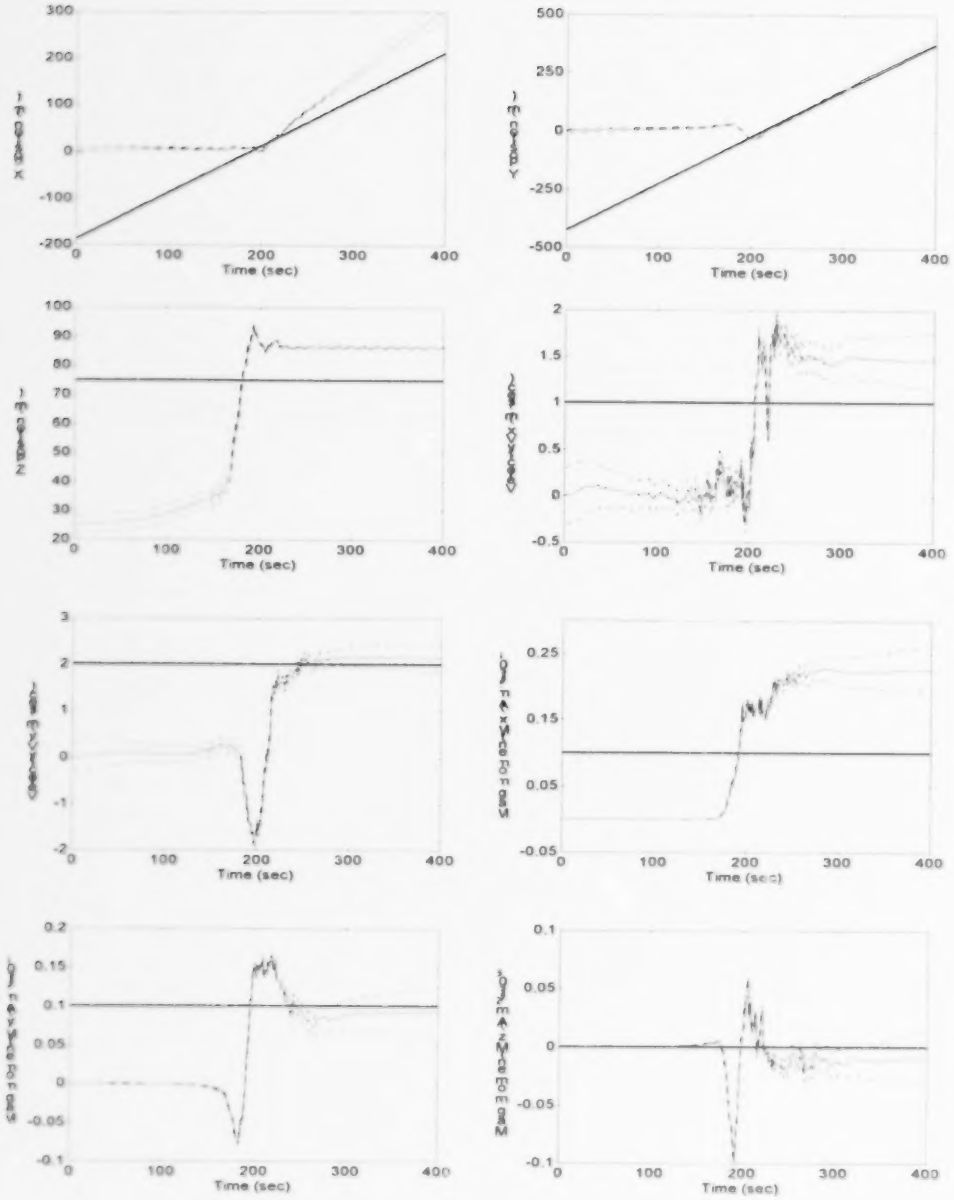

Figure 4. *State estimates from the UKF with ±3σ*

The simulation results favour the UKF, the ST1 and ST2 filters are intermediary, and the EKF algorithm diverges. The performance of the UKF is good, considering the absence of any information about the initial conditions and the high degree of non-linearity of the problem. Figure 3 shows the true values of state variables together with the state estimates obtained from this filter along with 3 times their standard deviations (dotted lines).

3. Conclusions

This report investigates the possibility of using non-linear Kalman filters for tracking and classification of a vessel modelled as an equivalent magnetic dipole of arbitrary orientation. The problem is formulated in state-space form where the (nine) state variables are the position, velocity and magnetic moment of the target. Because of the non-linearity of the observation equation, to solve this problem it is necessary to use various approximations to linearize the equation, which are known as the parametric non-linear Kalman filtering techniques. The filters used in this study are the extended Kalman filter (EKF), two Kalman filters based on the Stirling's first and second-order approximations (ST1 and ST2), and the unscented Kalman filter (UKF). All these filters are sub-optimal.

The filters are applied to a 2-observers situation that allows the complete tracking and classification problem to be solved. All the vessel parameters like the heading, speed, depth, and equivalent magnetic moments are estimated with a good level of accuracy by the UKF. From the tracking accuracy viewpoint, the EKF may diverge, but the other two filters (ST1 and ST2) appear to be statistically efficient. Because the filters starts at a point where the noisy magnetic signal from the target is very small (at the environmental level), they can be considered as permanently active. From this viewpoint, the proposed algorithm can be considered a detection method as well.

The performances of the four filters are evaluated using the absolute RMS errors obtained following 50 Monte Carlo simulations with different additional Gaussian noises. From the comparison of the RMS errors with the theoretical Cramer-Rao lower bound (CRLB) derived for this estimation problem, one can measure the effectiveness of each algorithm.

Compared to the previous study, the non-linear Kalman filtering technique has been extended to a 2-observers situation allowing the complete problem of tracking and classification to be solved. The fact that the direction of arrival is unknown makes this problem very challenging. New filters are implemented and tested. It is shown that the unscented Kalman filter (UKF) can perform successfully in estimating the states of the highly non-linear system. The state variables are correctly estimated and with reasonable precision given the small variance. Future work is necessary for testing the algorithms in real situations.

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Annexes

Implementing the Unscented Kalman Filter

The unscented transformation uses a set of weighted samples or 'sigma points', $S_i = \{W_i, \chi_i\}$, to completely capture the true mean and covariance of the random variable $\mathbf{x}$ and then propagates the sigma points through the non-linear function. We use the following notation: $\mathbf{K}$ is the Kalman gain, W_i are the weights, λ is the scaling parameter, $n_a = n_x + n_v + n_w$, and

$$\mathbf{x}^a = \left[\mathbf{x}^T \ \mathbf{v}^T \ \mathbf{w}^T\right]^T, \quad \mathcal{X}^a = \left[\left(\mathcal{X}^x\right)^T \left(\mathcal{X}^v\right)^T \left(\mathcal{X}^w\right)^T\right]^T$$

1. Initialize with:

$$\bar{\mathbf{x}}_0 = E\left[\mathbf{x}_0\right], \quad \mathbf{P}_0 = E\left[(\mathbf{x}_0 - \bar{\mathbf{x}}_0)(\mathbf{x}_0 - \bar{\mathbf{x}}_0)^T\right], \quad \bar{\mathbf{x}}_0^a = E\left[\mathbf{x}^a\right] = \left[\bar{\mathbf{x}}_0^T \ \mathbf{0} \ \mathbf{0}\right]^T$$

$$\mathbf{P}_0^a = E\left[(\mathbf{x}_0^a - \bar{\mathbf{x}}_0^a)(\mathbf{x}_0^a - \bar{\mathbf{x}}_0^a)^T\right] = \begin{bmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}$$

2. For discrete time samples, k = 1, 2, …, K,

   (a) Calculate sigma points:

$$\mathcal{X}_{k-1}^a = \left[\bar{\mathbf{x}}_{k-1}^a \quad \bar{\mathbf{x}}_{k-1}^a \pm \sqrt{(n_a + \lambda)\,\mathbf{P}_{k-1}^a}\right]$$

   (b) Time update:

$$\mathcal{X}_{k|k-1}^a = \mathbf{f}\left(\mathcal{X}_{k-1}^x, \mathcal{X}_{k-1}^v\right)$$

$$\bar{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n_a} W_i^{(m)} \mathcal{X}_{i,k|k-1}^x$$

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[\mathcal{X}_{i,k|k-1}^x - \bar{\mathbf{x}}_{k|k-1}\right]\left[\mathcal{X}_{i,k|k-1}^x - \bar{\mathbf{x}}_{k|k-1}\right]^T$$

$$\mathbf{Z}_{\mathbf{k}|\mathbf{k}-1} = \mathbf{h}\left(\aleph^{\mathbf{x}}_{\mathbf{k}|\mathbf{k}-1}, \aleph^{\mathbf{w}}_{\mathbf{k}-1}\right)$$

$$\bar{\mathbf{z}}_{\mathbf{k}|\mathbf{k}-1} = \sum_{i=0}^{2n_a} W_i^{(m)} \, Z_{i,k|k-1}$$

(c) Measurement update equations:

$$\mathbf{P}^{ZZ}_{k|k} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[Z_{i,k|k-1} - \bar{\mathbf{z}}_{\mathbf{k}|\mathbf{k}-1}\right]\left[Z_{i,k|k-1} - \bar{\mathbf{z}}_{\mathbf{k}|\mathbf{k}-1}\right]^T$$

$$\mathbf{P}^{XZ}_{k|k} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[\aleph_{i,k|k-1} - \bar{\mathbf{x}}_{\mathbf{k}|\mathbf{k}-1}\right]\left[Z_{i,k|k-1} - \bar{\mathbf{z}}_{\mathbf{k}|\mathbf{k}-1}\right]^T$$

Where $\mathbf{P^{zz}}$, $\mathbf{P^{xz}}$ are respectively the covariance matrix of the measurement and the cross-covariance of the measurement and the state variable. Next:

$$\mathbf{K}_\mathbf{k} = \mathbf{P}^{XZ}_{k|k}\left[\mathbf{P}^{ZZ}_{k|k}\right]^{-1}$$

$$\bar{\mathbf{x}}_\mathbf{k} = \bar{\mathbf{x}}_{\mathbf{k}|\mathbf{k}-1} + \mathbf{K}_\mathbf{k}\left(\mathbf{z}_\mathbf{k} - \bar{\mathbf{z}}_{\mathbf{k}|\mathbf{k}-1}\right)$$

$$\mathbf{P}_\mathbf{k} = \mathbf{P}_{\mathbf{k}|\mathbf{k}-1} - \mathbf{K}_\mathbf{k}\,\mathbf{P}^{ZZ}_{k|k}\,\mathbf{K}^\mathbf{T}_\mathbf{k}$$

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This report investigates the problem of tracking a vessel modeled at a distance by an equivalent magnetic dipole. Tracking a magnetic dipole from magnetic field measurements is a complex non-linear problem. The determination of target position, velocity and magnetic moment is formulated as an optimal stochastic estimation problem, which could be solved using the non-linear Kalman filtering methods. The estimation performance of the following filters is compared: the extended Kalman filter (EKF), two Kalman filters based on the Stirling's interpolation formula, and the unscented Kalman filter (UKF). To evaluate the algorithms, the theoretical Cramer-Rao lower bounds (CRLB) of estimation error are derived for this problem. Results obtained on simulated track show that the target position, velocity and moment can be accurately determined. It is shown that the unscented Kalman filter (UKF) is the best non-linear Kalman filter for this application.

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